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Magnetic Flow Meter Measurement of Solid Propellant Pressure-Coupled Responses Using an Acoustic Analysis

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Introduction

THE traditional methods for measuring solid propellant pressure-coupled responses such as the T-burner and modulated exhaust burner[1] are based on deducing the response by matching measured pressure oscillations, both amplitude and phase, with an acoustic analysis of the flow inside the test apparatus. The pressure-coupled response in the acoustic analysis is varied until the predicted pressure oscillations match the measured ones. These methods are indirect and are only as accurate as the acoustic analyses themselves.

Wilson and Micci[2] have developed a technique based on magnetic velocimetry for directly measuring a solid-propellant pressure-coupled response by simultaneously measuring the pressure and velocity oscillations (both amplitude and phase) in the combustion product gas above the surface of the burning solid propellant. The nondimensionalized ratio of the velocity to the pressure oscillations gives the complex acoustic admittance of the burning propellant surface from which both the real and imaginary components of the pressure-coupled response can be calculated. The technique developed by Wilson and Micci used the measured pressure and velocity oscillations immediately above the burning propellant surface as the propellant surface regressed past the measuring station. The data that was used to obtain the acoustic admittance was taken during a very small fraction of the total propellant strand burn time and several tests at each frequency of interest were required to obtain statistical confidence in the results.

This study used an improved version of the magnetic flow meter burner developed at ONERA Palaiseau Center combined with an acoustic analysis of the standing wave above the surface of the burning propellant strand to use velocity and pressure oscillation data taken from the surface of the propellant to as far as 1.2 cm above it, increasing the statistical confidence in the calculated acoustic admittance. The analysis itself is not required to obtain the propellant pressure-coupled response, shown by Wilson and Micci,[2] unlike the case with the T-burner and the modulated exhaust burner. Its validity is also ensured because it is applied only over a short distance above the burning propellant surface, minimizing heat losses that are not modeled. This analysis also allowed the simultaneous derivation of the magnetic flow meter calibration coefficient, eliminating the need for separate calibration tests.

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Experiment

The magnetic flow meter burner measures the velocity of the combustion product gas by applying a strong magnetic field (1860 G in this experiment) and measuring the strength of the electric field generated by the ionized combustion product gas moving through the magnetic field.[2] The electric field is equal to the cross product of the gas velocity and the magnetic field:

$$\boldsymbol{E} = \boldsymbol{u} \times \boldsymbol{B} \quad (1)$$

The electric field is measured by placing two electrodes in the periphery of the flow at right angles to both the magnetic field and the flow direction. The voltage measured is given by

$$V = \alpha u B l \quad (2)$$

where α is a nondimensional coefficient between 0.0–1.0 and l is the distance between the electrodes. The coefficient α is a function of a phenomenon known as end-shorting caused by a nonspatially uniform magnetic field[3] and must be determined experimentally. The magnetic field was generated by a permanent magnet. Both mean and oscillatory pressures were measured with a piezoelectric transducer. The propellant strands were cylinders that end burned on the flat surface. A pressure oscillation at the frequency of interest was generated

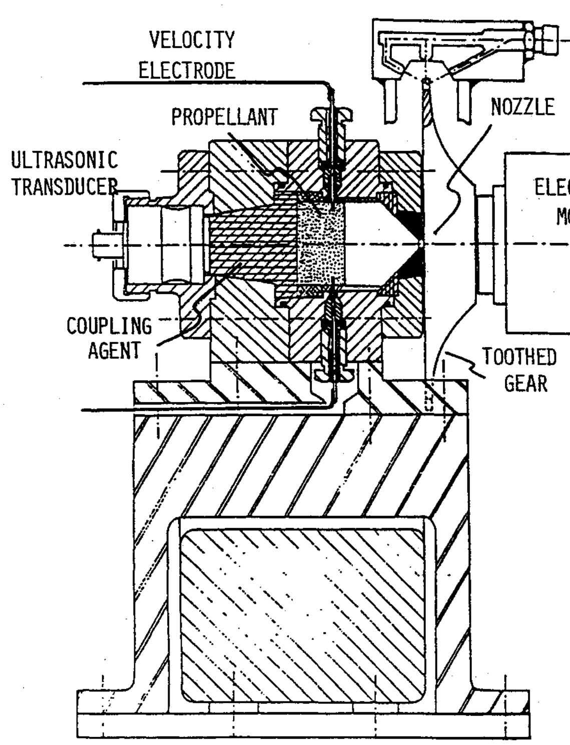

Fig. 1 Magnetic flow meter burner viewed perpendicular to the axis of the motor showing the installation of the velocity electrodes and the ultrasonic transducer.

by means of a toothed wheel rotated over the exit of the converging graphite nozzle.

The two improvements in the magnetic flow meter burner were the use of a larger-diameter propellant strand and an ultrasound transducer (Fig. 1). The larger-diameter propellant strand allowed the use of much larger throat diameters, reducing the risk of aborted runs because of overpressurization caused by nozzle blockage, particularly for metallized propellants. The ultrasound transducer[4] determined the location of the burning propellant surface by measuring the time delay of an ultrasound pulse transmitted through the unburned propellant and reflected by the burning propellant surface. This allowed the measurement of the distance between the axial location of the electrodes and the burning propellant surface simultaneously with the pressure and velocity measurements. These measurements combined with the following acoustic analysis allow the use of data taken over a period of time while the electrodes are in close proximity to the propellant surface. The mean and oscillatory pressures, oscillatory velocity, distance burned by means of the ultrasound transducer, and the modulation frequency were digitized and recorded on a HP 9000 computer.

Analysis

The assumptions made in deriving the acoustic analysis are as follows:

1) Both the mean flow and the acoustic oscillations are one dimensional in a direction normal to the propellant surface and the combustion product gas obeys the perfect gas law.

2) The geometry of the combustion chamber is a constant area cylinder, therefore, the mean flow properties are constant from the propellant surface to the entrance of the nozzle.

3) The mean flow properties, frequency of oscillation, burning propellant surface area, magnetic flow meter calibration constant, and propellant acoustic admittance are constant during a test.

4) The force generated by the magnetic field on the ionized combustion products does not influence either the mean or oscillatory flowfields.

5) The magnitudes of the oscillatory flow properties are much less than their mean values and are isentropic.

Using the small perturbation form of the equations for the conservation of mass, momentum, and energy, and writing all variables as the sum of a mean and a harmonically oscillating part, i.e.,

$$f = \bar{f} + f' e^{i\omega t} \tag{3}$$

one obtains the linear one-dimensional equations with mean flow for the oscillatory components of pressure and velocity:

$$p'(x) = \{Y \exp[ikx/(1 - \bar{M})] + Z \exp[-ikx/(1 + \bar{M})]\}e^{i\omega t} \tag{4}$$

$$u'(x) = (1/\bar{\rho}\bar{a})\{-Y \exp[ikx/(1 - \bar{M})] + Z \exp[-ikx/(1 + \bar{M})]\}e^{i\omega t} \tag{5}$$

where $k = \omega/\bar{a}$ and $\bar{M}$ is the Mach number of the mean flow.

The acoustic admittance as a function of x is

$$A(x) = \bar{\rho}\bar{a}[u'(x)/p'(x)] \tag{6}$$

where the acoustic admittance of the burning propellant surface A_b is $A(x = 0)$. Combining Eqs. (4) and (5), setting $x = 0$, and substituting in A_b gives

$$Z = Y[(1 + A_b)/(1 - A_b)] \tag{7}$$

Substituting Eq. (7) into Eqs. (4) and (5) and taking the ratio of the two equations to obtain the admittance as given by Eq. (6) yields

$$A_b = \frac{A(X)\cdot(X + 1) + X - 1}{A(X)\cdot(X - 1) + X + 1} \tag{8}$$

where

$$X = \exp[2ikx/(1 - \bar{M}^2)] \tag{9}$$

The experiment gives, at each sampling time, the complex pressure oscillation p', the distance between the measurement station and the burning propellant surface x, the mean chamber pressure $\bar{p}$, the modulation frequency, and the complex velocity oscillation times the magnetic flow meter calibration coefficient $\alpha u'$. Thus, at any time during the firing one can calculate both $X(x)$ and $\alpha A(x)$ and form for each measurement j, $j = 1$ to N:

$$A_b(X_j + 1) - C(\alpha A)_j(X_j + 1) + CA_b(\alpha A)_j(X_j - 1) = X_j - 1 \tag{10}$$

where $C = 1/\alpha$.

The system of N nonlinear equations is linearized by means of the Newton–Raphson technique and the matrix is solved iteratively by a least-squares method until convergence is achieved to obtain values for A_b and C. The coefficient α is obtained from $1/C$ and the propellant pressure-coupled response R_p is obtained from

$$R_p = (A_b/\gamma\bar{M}) + 1 \tag{11}$$

Results

Three composite solid propellant formulations using ammonium perchlorate as the oxidizer were tested: 1) Butalite 80, a nonmetallized propellant formulated by ONERA; 2) the Ariane V solid rocket booster propellant; and 3) NWR-11 from Naval Weapons Center (NWC), China Lake, California. Two tests conducted with the Ariane V propellant were not successful because of electrical problems with the pressure and velocity instrumentation unrelated to the propellant formulation. Because of environmental and experimental problems, both of which were solved, only two of the six tests with Butalite 80 resulted in sufficiently good signal quality to produce realistic response values.

NWR-11 is a propellant formulated for pressure-coupled response measurement and comparison by several investigators in both the U.S. and France. Eight tests were conducted with the magnetic flow meter burner at a mean chamber pressure of 35 bar. Because of experimental problems, only four tests could be analyzed in terms of responses. This resulted in one measurement at each of the following frequencies: 300, 500, 600, and 1040 Hz. Figure 2 plots the real part of the response $Re(R_p)$ vs frequency. The value plotted at zero frequency is the propellant burning rate exponent, 0.49, and the drawn line is a third-order polynomial least-squares fit to the magnetic flow meter burner data. Also shown on Fig. 2 are response values at 35 bar for the same propellant obtained by ONERA with

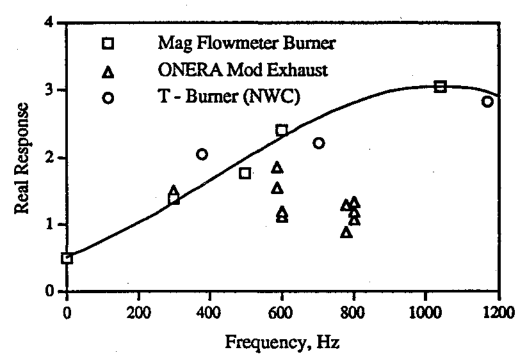

Fig. 2 Real part of the pressure-coupled response as a function of frequency for NWR-11 propellant as measured by the magnetic flowmeter, ONERA modulated exhaust motor, and NWC T-burner.

the modulated exhaust motor[5] and by NWC with the T-burner.[6] It can be seen that the magnetic flow meter burner gives results similar to the T-burner and, in fact, gives a better fit to a standard response function curve. With NWR-11 a very noisy $u(t)$ signal was observed, resulting in noisy $u'(t)$ signals where the forced oscillation is only weakly visible. This led to a lower quality in the data reduction in terms of R_p and α. This may be because NWR-11 contains a large portion of coarse AP (50% by weight of 400 μm), which may result in increased flow turbulence near the propellant surface registering as broadband noise by the magnetic flow meter.

Conclusions

The addition of an ultrasound transducer for measuring the location of the propellant surface relative to the velocity measurement station and the use of a larger diameter propellant strand improved the ability of the magnetic flow meter to provide accurate data for the determination of a solid propellant pressure-coupled response. The use of an acoustic analysis in combination with the measurements allowed the use of data taken over a period of time, improving the accuracy of the calculated response and improving repeatability.

Acknowledgments

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Magnetohydrodynamics of a Particulate Suspension

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Introduction

CHAMKHA[1] reported exact solutions for the steady hydromagnetic two-dimensional flow of a particle-fluid suspension past an infinite porous flat plate. In the model used, the particle phase was assumed stress-free. The purpose of this Note is to generalize the model used by Chamkha[1] to include particle-phase viscous effects.

The mathematical model employed herein represents a generalization of the original dusty-gas model discussed by Marble[2] to include particle-phase viscosity and magnetic field effects. Particle-phase viscosity can be used to model several effects. Among these are particle-particle interactions and Reynolds stresses resulting from using a continuum model to represent a cloud of discrete particles.[3]

Governing Equations

Let x denote the coordinate parallel to the direction of the flow, and y the coordinate perpendicular to it. Let a uniform magnetic field be applied along the y axis. Far from the plate both phases are in equilibrium and moving with speed V_∞ in the x direction. At the plate surface uniform suction with speed V_0 is applied to the fluid phase. The fluid phase is assumed incompressible and electrically conducting. The suspended particles are assumed electrically nonconducting and have a small volume fraction. In addition, the magnetic Reynolds number is assumed to be small and the induced magnetic field is neglected.

The modified dusty-gas model to be employed herein can be written as

$$\begin{aligned} -V_0\partial_y u &= \nu\partial_{yy}u + N\rho_p/\rho(u_p - u) + \sigma B_0^2/\rho(V_\infty - u) \\ -V_0\partial_y u_p &= \nu_p\partial_{yy}u_p + N(u - u_p) \end{aligned} \tag{1}$$

where ρ, u, and ν are the fluid-phase density, velocity in the x direction, and kinematic viscosity, respectively. ρ_p, u_p, and ν_p are the particulate-phase insuspension density (mass of particles per unit volume of suspension), velocity in the x direction, and kinematic viscosity, respectively. σ, B_0, and N are the electrical conductivity, the magnetic induction, and the interphase force coefficient, respectively. In the present work the coefficients ρ, ρ_p, ν, ν_p, N, and B_0 will all be treated as constants.

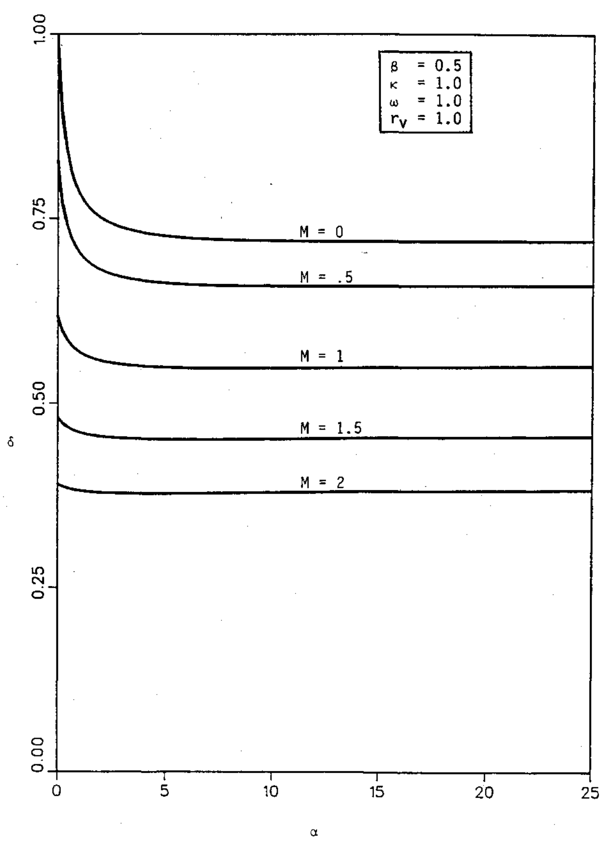

Fig. 1 Fluid-phase displacement thickness vs α.

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